Design Aid for 5-layer Cross Laminated Timber Plate Under 3-side Support

Yunbo Huang

June 21, 2021

Contents

1 Abstract

Deflection of cross laminated timber (CLT) plate under 3-side simply support is difficult to hand-calculate due to the anisotropy of wood. By Finite Element Modeling (FEM), maximum deflections of 5-layer CLT plates with different lengths, width, layer thickness combinations and strength combinations can be acquired. The relations between different combinations of CLT plates and their maximum deflections were built by non-linear curve fitting processes and shown in the form of an Excel sheet. The method can be used to obtain deflections of any plate combination within the preset range, with an accuracy of 1 mm for most cases.

2 Introduction

Construction of wooden multi-story buildings is increasing and is an important part of northern Sweden. Many dwellings, especially high-rise dwellings or blocks of flats, are now provided with a balcony. Among those balconies, some of them are made of CLT plates.

To design CLT balcony, one of the issues that should be addressed is deflection. The commonly adopted procedure of obtaining the maximum deflection is by simulation in FEM software. Creating a FEM model and run the simulation takes time and requires expertise in the method and structural design. A simple tool to hand-calculate maximum deflections of CLT plates eases the design process.

Few studies have been conducted with regard to the flexural property of CLT plates. Wang et al .(2020) [2] studied the bending properties of CLT plate under 4-side simply support through FEM simulations and experimental tests. In the case of CLT plate under 3-side support, no studies of deflection-calculation method has been found.

Based on FEM software, a large number of CLT plate combinations and the corresponding deflections are obtained. This report presented a numerical way of building the connections between different combinations and the relating deflections by non-linear curve fitting process.

3 List of variables

Table 1: Explanation of variable symbols used in this report

Symbol	Meaning
L	Length of the CLT plate, in m
B	Width of the CLT plate, in m
T	Total thickness of the CLT plate, all thicknesses are in mm
T1	Thickness of layer 1
T2	Thickness of layer 2
T3	Thickness of layer 3
T4	Thickness of layer 4
T5	Thickness of layer 5
Ex	Modulus of elasticity in X direction, in GPa
Ey	Modulus of elasticity in Y direction,in GPa
EIx	Modulus of elasticity in Y direction times moment of innertia in Y direction, in 10E+11 $N \cdot mm^2$
EIy	Modulus of elasticity in X direction times moment of innertia in X direction, in 10E+11 $N \cdot mm^2$
$a_0 - a_4$	Coefficent for equation model: $a \cdot x^b + c$
$b_0 - b_4$	Coefficent for equation model: $a \cdot x^b + c$
$c_0 - c_4$	Coefficent for equation model: $a \cdot x^b + c$
$p_1 - p_2$	Coefficent for equation model: $p_1 \cdot x + p_2$
$W_{6\times1.2}$	Deflection for size $6 \times 1.2\, m^2$, all deflections are in mm
$W_{6\times B}$	Deflection for length of 6 m
$W_{1.2}$	Deflection for width of 1.2 m
W_6	Deflection for length of 6 m

4 Method

4.1 Assumptions

1. For a 5-layer 3-side simply supported CLT plate, parameters (variables) that have main effects on the maximum deflections are assumed to be: length, width, layer thickness combinations and strength combinations, see figure 1

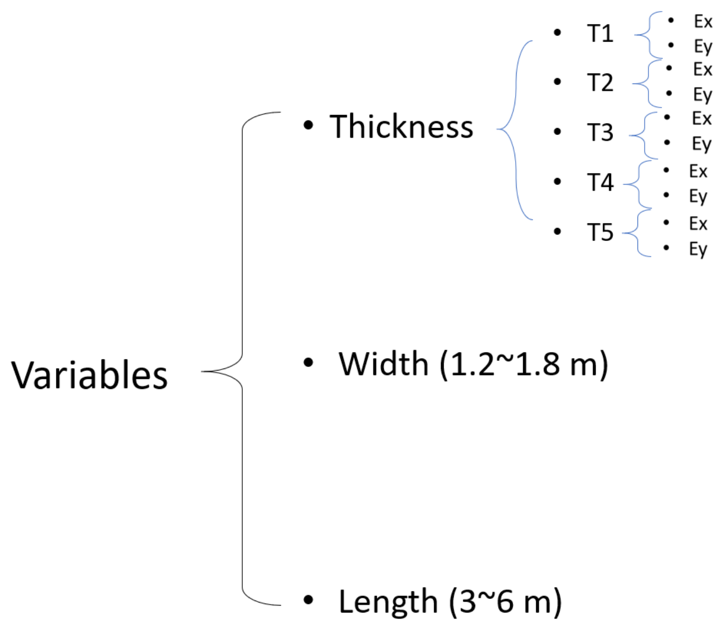


Figure 1: 4 main variables taken into account in the project.Length and width range were decided based on commonly seen size of CLT plate without intermediate support.

2. Even though it is difficult to hand-calculate the deflections of CLT plate due to the anisotropic properties of wood material, there are certain connections between a specific plate and its corresponding deflection. That is to say, the maximum deflection of a CLT plate with determined variables is unique.

   In a common case, those connections are used as formulas to calculate deflections based on plate parameters. If plate parameters and corresponding maximum deformations are already known, it is possible to figure out how those parameters are related to the corresponding formulas, provided that a large number of cases are analyzed, see figure 2.

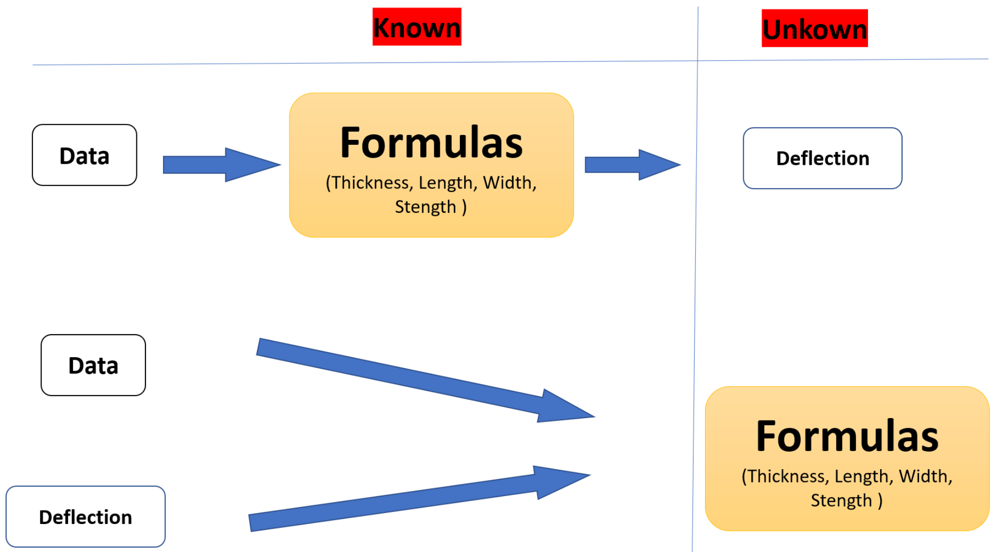


Figure 2: The relations between data, deflection and formulas.

3. In the real case, the plate is pinned lengthwise to the wall with two point- support at 1.2[1] meters from the wall (figure 3b). For an easy analysis, the support was assumed as 3-side support (figure 3c).

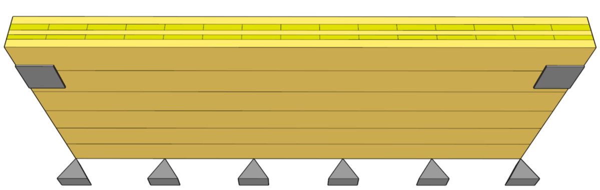
4. Distributed load is assumed to be the cause of deflections. Point load is not considered in this project. The magnitude of the load is another variable, but it is linearly related to the deflections.

4.2 List of representative cases

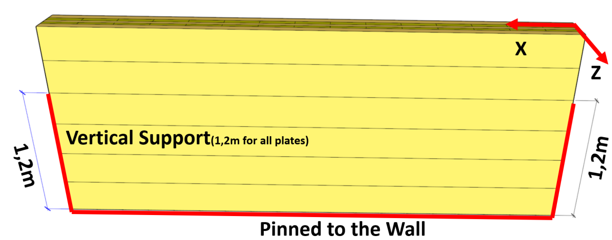
1. 5-layer CLT plates are normally 100-200 mm thick. 4 different thicknesses were chosen for analysis: 140 mm, 150 mm, 170 mm, 200 mm (figure 4b).
2. Under each thickness, more than 4 different layer combinations were chosen (figure 4a).
3. For each layer combination, 5 strength classes were analyzed.
4. The width range of plates was determined as 1.2-1.8 m, while the length was 3-6 m (figure 4a).
5. The magnitude of distributed load was set as 3 kN/m^2.

[1]"." is used in the report as decimal separator.

(a) Rendered picture of the real balcony. The wall-side is fixed by steel brackets, the other two sides are supported by steel rods [1].

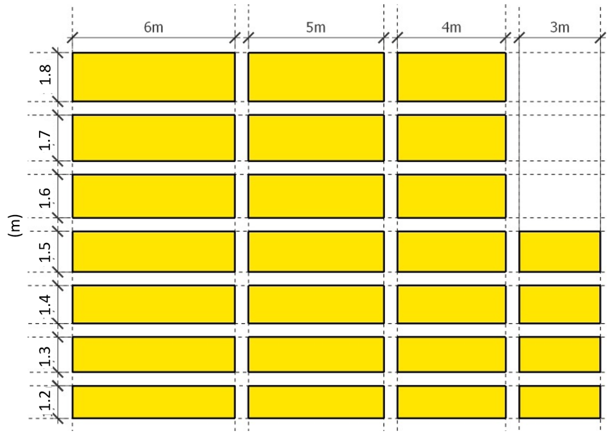


(b) The real support based on 3a.

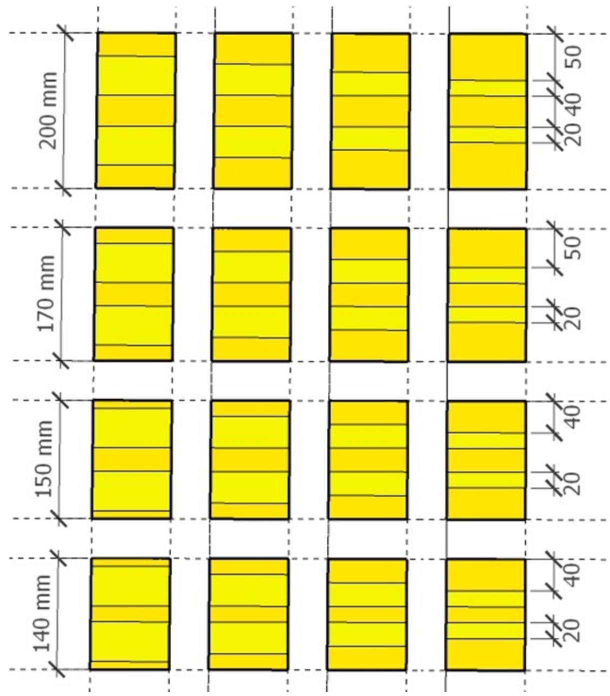


(c) The ideal support

Figure 3: The actual support and the ideal support.

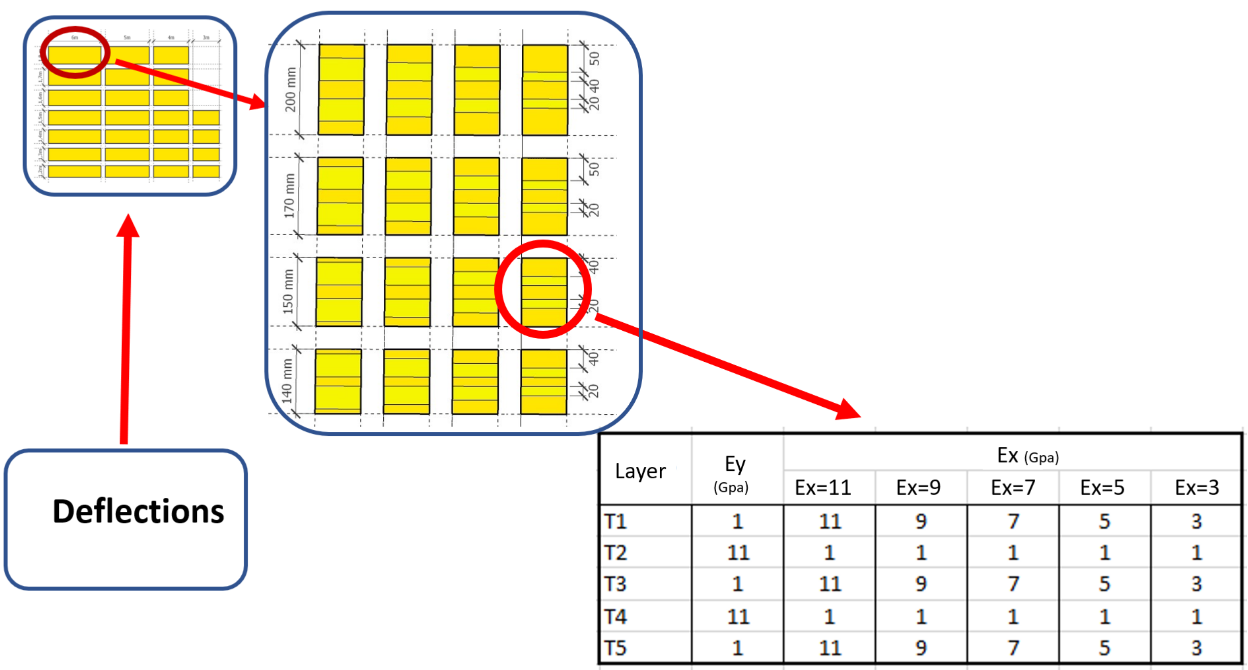


(a) Representative length and width combinations.



(b) Representative thicknesses and layer combinations.

Figure 4: Part of the cases chosen for the connection-building process.



Layer	Ey (Gpa)	Ex (Gpa)				
		Ex=11	Ex=9	Ex=7	Ex=5	Ex=3
T1	1	11	9	7	5	3
T2	11	1	1	1	1	1
T3	1	11	9	7	5	3
T4	11	1	1	1	1	1
T5	1	11	9	7	5	3

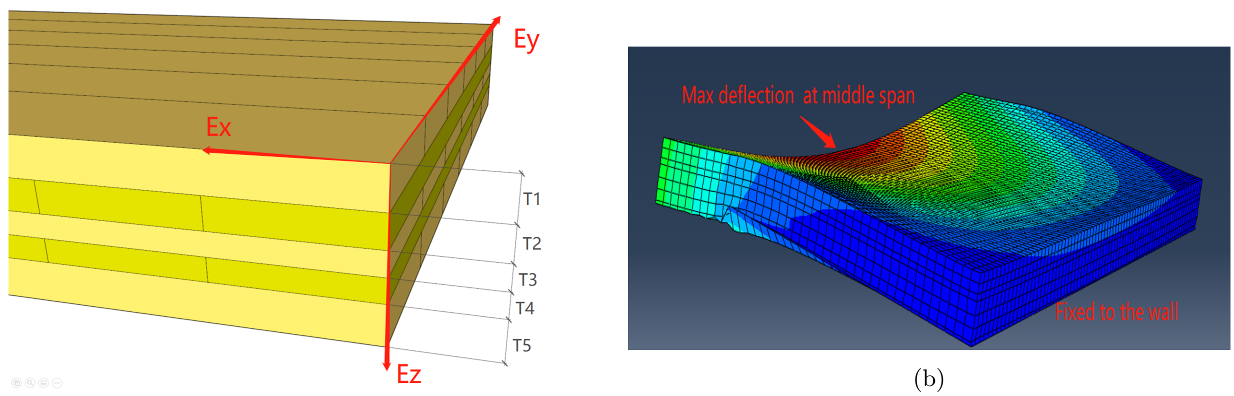
Figure 5: Illustration of how those design representatives are related to each other (definition of layers and orientation is seen in figure 6a.).

4.3 Simulation by Finite Element Modeling

Finite Element modeling software Abaqus was used to obtain maximum deflections for each CLT combination. The fixture at the wall-side (lengthwise) was set as pinned, while the other two sides only had vertical supports. Mechanical properties of the plate were set as in table 2. Mesh size was set to be 0.004 m as a balance between simulation time and level of accuracy, example of FEM simulation is seen in figure 6b.

Table 2: Mechanical properties of CLT plate in Abaqus. Ex and Ey may vary depending on the strength combinations, the rest parameters remain the same throughout the process. U is Poisson's ratio, G is shear modulus in GPa.

Ex	Ey	Ez	Uxy	Uxz	Uyz	Gxy	Gxz	Gyz
11	1	0.3	0.45	0.45	0.3	0.65	0.65	0.05



(a) (b)

Figure 6: Definition of layers and orientation of CLT plate, X-direction is in lengthwise.

4.4 Data processing

Once all data had been recorded, the next thing was to build connections. It was found that, when length was 6 m (L=6 m), deflection at width 1.2 m (B = 1.2 m) could be used as the base for other widths.

4.4.1 Deflection for size $6 \times 1.2\, m^2$

Different layer thickness combinations and strength combinations affect the deflection in the form of EI_x and EI_y. In terms of deflection for size $6 \times 1.2\, m^2$, EI_y had a minor effect, which was not taken into consideration when building the connections. Strength and layer combinations were taken into consideration in terms of EI_X, making it the only variable here in the table. Explanation of the strength distribution can be seen in figure 5. Each CLT plate has a ID number, see table 13 in Appendix section A.

Based on the table 3, deflection for size $6 \times 1.2\, m^2$ ($W_{6\times1.2}$) can be connected to EI_X with the help of Matlab by using a non-linear curve fitting tool, which can be expressed as (see figure 13 in Appendix):

$$W_{6\times1.2} = a_0 \cdot \sqrt{EIx} + c_0 \tag{1}$$

a_0 and c_0 are coefficients for the equation, with no units. For different thicknesses, a_0 and c_0 are different, see table 4. A further connection between a_0 and T, c_0 and T can be expressed as:

Table 3: Example of FEM results when T=140 mm.

CLT No.	W1.2	W1.3	W1.4	W1.5	W1.6	W1.7	W1.8	Ex	EIx
22	14.07	15.57	17.23	19.02	20.95	23.04	25.32	11	20.95
22	14.97	16.65	18.52	20.54	22.27	25.07	27.63	9	17.22
22	16.04	17.95	20.08	22.4	24.9	27.61	30.55	7	13.49
22	17.33	19.53	22.02	24.73	27.68	30.88	34.35	5	9.75
22	18.94	21.56	24.52	27.79	31.38	35.3	39.57	3	6.02
22	21.22	24.41	28.1	32.24	36.86	41.97	47.61	1	2.29
20	17.51	19.6	21.9	24.36	26.96	29.71	32.61	11	10.82
20	18.22	20.47	22.96	25.63	28.46	31.47	34.65	9	9.11
20	19.02	21.45	24.15	27.08	30.19	33.51	37.03	7	7.41
20	19.93	22.58	25.55	28.76	32.21	35.91	39.86	5	5.70
20	20.99	23.9	27.19	30.77	34.65	38.84	43.34	3	3.99
1	15.23	16.91	18.75	20.7	22.79	25.03	27.41	11	16.88
1	16.1	17.94	19.99	22.18	24.51	27.01	29.67	9	13.96
1	17.09	19.16	21.45	23.93	26.58	29.41	32.44	7	11.04
1	18.27	20.6	23.22	26.07	29.13	32.42	35.95	5	8.12
1	19.71	22.39	25.44	28.78	32.4	36.33	40.58	3	5.20

Table 4: a_0 and c_0 values for different thicknesses.

T(mm)	a_0	c_0
140	-2.512	25.52
150	-1.88	20.99
170	-1.17	15.31
200	-0.5987	10.19

$$a_0 = \frac{-74,69E+7}{T^{3,949}} \tag{2}$$

$$c_0 = \frac{73,86E+5}{T^{2,546}} \tag{3}$$

To compile, the formula to alculate the maximum deflection for 5-layer CLT plate of any layer combinations and strength combinations is obtained:

$$W_{6\times1.2} = \frac{-74,69E+7}{T^{3.949}} \cdot \sqrt{EIx} + \frac{73.86E+5}{T^{2.546}} \tag{4}$$

4.4.2 Deflection for length of $6\,m$

Deflection for length of 6 m ($W_{6\times B}$) was based on $W_{6\times1.2}$. Based on table 3, the ratio k_B ($k_{1.2}$,$k_{1.3}$,etc) between $W_{6\times B}$ and $W_{6\times1.2}$ ($W_{6\times B}$ / $W_{6\times1.2}$) was derived, as shown in table 5.

It was found that K_B and (B-1.2) can be linked in a good way:

$$K_B = e^{(B-1.2)\cdot b_1} \tag{5}$$

In which b_1 is seen in table 5 as well. It can be seen that b_1 is dependent on EI_x and EI_y within 140 mm. The connection between b_1 and EI_x within the same EI_y is:

$$b_1 = a_2 \cdot EIx^{b_2} \tag{6}$$

Table 5: ration between $W_{6\times B}$ and $W_{6\times 1.2}$, and value for b_1 when T=140 mm.

CLT No.	k1.2	k1.3	k1.4	k1.5	k1.6	k1.7	k1.8	Ex	EIx	EIy	**b1**
22	1.000	1.107	1.225	1.352	1.489	1.638	1.800	11	20.9526	6.480	0.9862
22	1.000	1.112	1.237	1.372	1.488	1.675	1.846	9	17.2194		1.023
22	1.000	1.119	1.252	1.397	1.552	1.721	1.905	7	13.4862		1.085
22	1.000	1.127	1.271	1.427	1.597	1.782	1.982	5	9.753		1.153
22	1.000	1.138	1.295	1.467	1.657	1.864	2.089	3	6.0198		1.242
20	1.000	1.119	1.251	1.391	1.540	1.697	1.862	11	10.8196	16.620	1.055
20	1.000	1.123	1.260	1.407	1.562	1.727	1.902	9	9.113		1.09
20	1.000	1.128	1.270	1.424	1.587	1.762	1.947	7	7.4064		1.129
20	1.000	1.133	1.282	1.443	1.616	1.802	2.000	5	5.6998		1.174
20	1.000	1.139	1.295	1.466	1.651	1.850	2.065	3	3.9932		1.227
1	1.309	1.483	1.678	1.888	2.115	2.358	2.617	11	16.88	10.53	0.9921
1	1.000	1.114	1.242	1.378	1.522	1.678	1.843	9	13.96		1.033
1	1.000	1.121	1.255	1.400	1.555	1.721	1.898	7	11.04		1.084
1	1.000	1.128	1.271	1.427	1.594	1.774	1.968	5	8.12		1.144
1	1.000	1.136	1.291	1.460	1.644	1.843	2.059	3	5.2		1.22

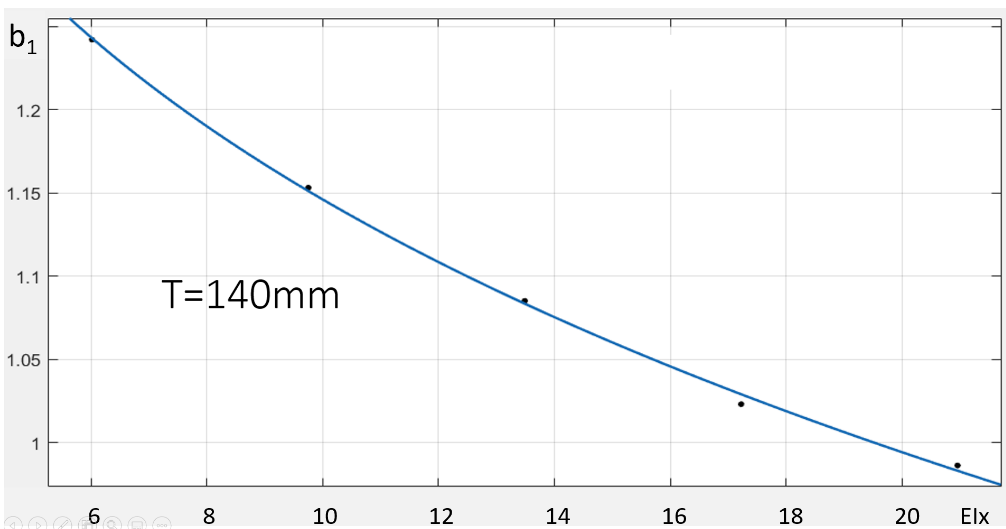


Figure 7: The curve fitting process for equation 6. X-axis is EI_x and y-axis is b_1.

Here, a_2 and b_2 are variables that depend on EI_y within the same thickness. A compilation of a_2 and b_2 is seen in table 6.

Table 6: A compilation of a_2 and b_2 across the thicknesses.

T(mm)	**CLT** No.	**a2**	**b2**	**EIy**
140	22	1.742	-0.1852	6.49
	20	1.516	-0.1498	16.62
	1	1.634	-0.1736	10.55
150	32	1.599	-0.139	20.90
	31	1.688	-0.1657	13.68
	30	1.783	-0.1746	8.66
	33	1.944	-0.1721	3.63
170	5	1.902	-0.1679	9.94
	9	1.575	-0.1298	31.99
	10	1.978	-0.1537	4.91
	7	1.973	-0.166	6.73
200	40	1.71	-0.1317	40.27
	6	1.924	-0.1483	20.53
	43	2.152	-0.1613	10.40

a_2 and b_2 can be expressed as:

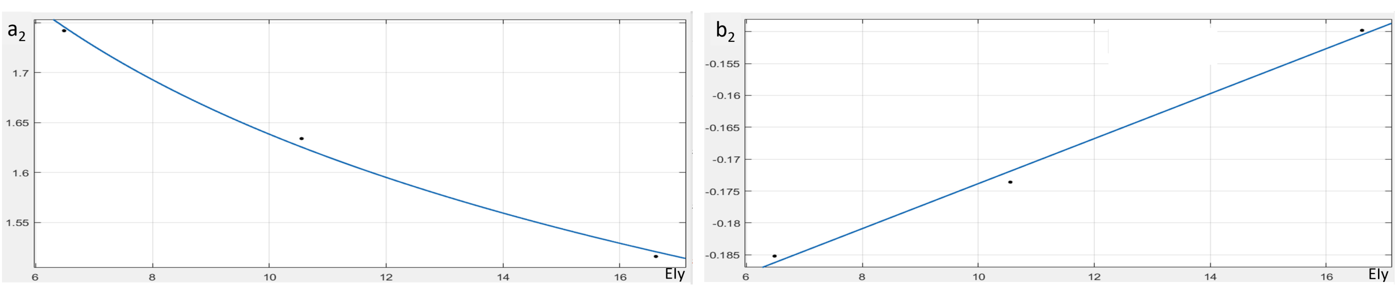
$$a_2 = a_3 \cdot EIy^{-0.15} \tag{7}$$

$$b_2 = p_1 \cdot EIy + p_2 \tag{8}$$

Where p_1 and p_2 are coefficients for polynomial equation model, without units. When T=140 mm, a_2 and b_2 are:

$$a_2 = 2.134 \cdot EIy^{-0.15} \tag{9}$$

$$b_2 = 0.003527 \cdot EIy - 0.2091 \tag{10}$$



(a) (b)

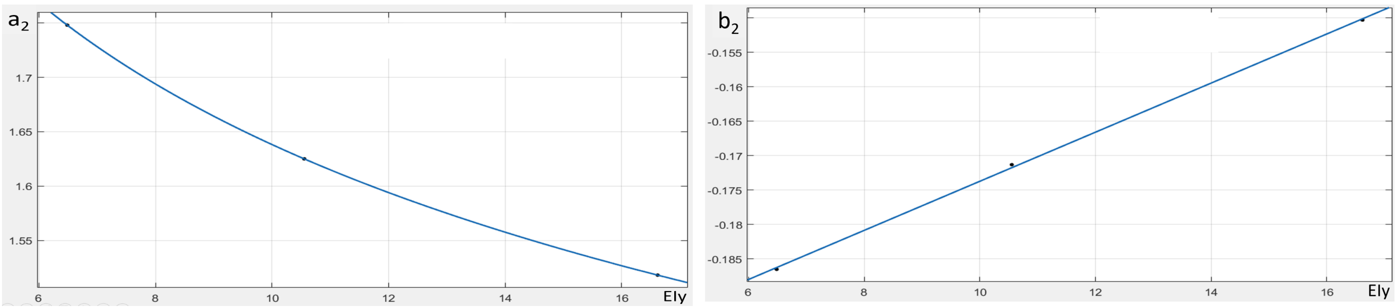
Figure 8: Curve-fitting for Equation 9 and Equation 10.

It can be seen from figure 8 that the curves were not perfectly fitted. By Equation 9 and EI_y in table 6, a new set of a_2 values are obtained, seen in table 7.

Table 7: a_2 and b_2 values after tuning when T=140 mm. The values can be compared with those in table 6.

a2	**b2**
1.748075	-0.1865
1.517989	-0.1503
1.625006	-0.1713

a_2 and b_2 were derived from Equation 6, meaning that any changes in either one will affect the other. Using the a_2 values in table 7 to fit the curve as shown in figure 7 will lead to unique b_2 value in table 7.



(a) (b)

Figure 9: Curve-fitting process for a_2 and b_2 after tuning. Compared to figure 8, the curves are better-fitted.

a_2 is dependent on a_3 (see Equation 7), b_2 is dependent on p_1 and p_2 (see figure 8). a_3, p_1 and p_2 vary across the thicknesses. A compilation of values is seen in table 8.

Table 8: Values for a_3, p_1 and p_2.

T(mm)	**a2**		**b2**	
	a3	b3	p1	p2
140	2.314	-0.15	0.003601	-0.2102
150	2.4198	-0.15	0.00289	-0.1921
170	2.652	-0.15	0.001502	-0.1781
200	3.027	-0.15	0.001021	-0.1689

Based on table 8, connections can be built:

$$a_3 = 0.0003689 \cdot T^{1.586} + 1.377 \tag{11}$$

$$p_1 = (8.752E - 07) \cdot T^2 - 0.0003422 \cdot T + 0.03444 \tag{12}$$

$$p_2 = (-4.906E + 12) \cdot T^{-6.541} - 0.1647 \tag{13}$$

By the formulas derived, deformation of any CLT plate under size $6\,m \times 1.2\,m - 1.8\,m$ ($W_{6\times B}$), regardless of the thickness and strength class, can be calculated. The next thing is to find formulas for other lengths and widths.

4.4.3 Deflection for size $5 \times 1.2\,m^2$

$W_{6\times B}$ can be used to build connections with $W_{5\times B}$, as shown in table 9

For $W_{5\times B}$, take $W_{5\times 1.2}$ as an representative for the whole width range. A modification factor will be used for the widths other than 1.2 m.

Table 9: An example of deflections for CLT No.22 at different sizes. The connections were build by the ratio of $W_{5\times B}$ to $W_{6\times B}$, and $W_{4\times B}$ to $W_{6\times B}$. $W_{1.2}$ - $W_{1.8}$ are in mm.

L (m)	W1.2	W1.3	W1.4	W1.5	W1.6	W1.7	W1.8
6	11.16	12.38	13.74	15.23	16.86	18.66	20.66
5	6.93	7.62	8.41	9.29	10.29	11.44	12.76
4	3.74	4.07	4.47	4.94	5.53	6.23	7.1
W6/W6	1	1	1	1	1	1	1
W5/W6	0.62	0.62	0.61	0.61	0.61	0.61	0.62
W4/W6	0.34	0.33	0.33	0.32	0.33	0.33	0.34

Table 10: An example of ($W_{5\times1.2}/W_{6\times1.2}$)and corresponding parameters when T=170 mm

CLT No.	**Ex**	**EIx**	**EIy**	$W_{5\times1.2}/W_{6\times1.2}$
3	11	34.169	14.960	0.631
3	9	28.154		0.637
3	7	22.139		0.646
3	5	16.124		0.657
3	3	10.109		0.671
5	11	39.186	9.944	0.625
5	9	32.168		0.633
5	7	25.149		0.642
5	5	18.131		0.653
5	3	11.113		0.669
7	11	42.403	6.728	0.623
7	9	34.741		0.630
7	7	27.079		0.640
7	5	19.418		0.653
7	3	11.756		0.669
8	11	26.953	22.180	0.637
8	9	22.381		0.643
8	7	17.809		0.651
8	5	13.238		0.661
8	3	8.666		0.673

The connections can be expressed between ($W_{5\times1.2}/W_{6\times1.2}$) and EI_x from table 10, without considering EI_y:

$$W_{5\times1.2}/W_{6\times1.2} = a_4 \cdot EIx^{b_4} \tag{14}$$

Table 11: Compilation of a_4 and b_4 across the thicknesses. V1 means before tuning and V2 after tuning.

	V1		V2	
T(mm)	**a4**	**b4**	**a4**	**b4**
170	0.7556	-0.0511	0.7556	-0.0511
140	0.7243	-0.0487	0.7244	-0.0487
150	0.7353	-0.0492	0.7354	-0.0492
200	0.7728	-0.0490	0.7924	-0.0562

From table 11, a_4 and b_4 are:

$$a_4 = 0.000745 \cdot T + 0.625 \tag{15}$$

$$b_4 = 0.0001917 \cdot T - 0.05039 \tag{16}$$

To put formulas together, $W_{5\times1.2}$ can be expressed as:

$$W_{5\times1.2} = W_{6\times1.2} \cdot K_{5\times1.2} \tag{17}$$

$$W_{5\times1.2} = W_{6\times1.2} \cdot (0.000745 \cdot T + 0.625) \cdot EIx^{0.0001917\cdot T-0.05039} \tag{18}$$

4.4.4 Deflection for length of $5\,m$

From table 14 in Appendix, $K_{5\times B}$ is tuned as:

$$K_{5\times B} = K_{5\times1.2} - 0.008/0.6 \cdot (B - 1.2) \tag{19}$$

$$W_{5\times B} = W_{6\times B} \cdot ((0.000745 \cdot T + 0.625) \cdot EIx^{0.0001917\cdot T-0.05039} - 0.008/0.6 \cdot (B - 1.2)) \tag{20}$$

4.4.5 Deflection for length of $4\,m$ and $3\,m$

Applying the same method for $W_{5\times B}$, $W_{4\times B}$ and $W_{3\times B}$ are derived:

$$W_{4\times B} = W_{6\times B} \cdot (0.001125 \cdot T + 0.3395) \cdot EIx^{0.0001605\cdot T-0.1471} \tag{21}$$

$$W_{3\times B} = W_{6\times B} \cdot (0.000976 \cdot T + 0.162) \cdot EIx^{0.000656\cdot T-0.3269} \tag{22}$$

Equation 22 only calculates CLT plate with a width less than 1.5 m.

4.4.6 Continuous length

Based on Equation 20, 21 and 22, deflections any CLT plates at length 6,5,4 and 3 meters can be calculated at any width within 1.2-1.8 m. However, those formulas do not work when lengths are not integers.

Take $W_{4.5\times1.55}$ as an example. It is possible to get $W_{5\times1.55}$ by Equation 20 and $W_{4\times1.55}$ by Equation 21. $W_{4.5\times1.55}$ can be obtained by interpolating.

5 Results

All formulas derived in section 4.4 are transferred to a Excel sheet, to make hand-calculation of maximum deflections possible by simply input plate parameters. The sheet is divided into 4 blocks: input data, results, status and intermediate parameters, see figure 10. Two figures in the sheet help to define all 5 layers, orientation of the plate and type of support.

5.1 Input data

As being discussed, there are mainly 4 variable that will affect deflection of the plate, plus load magnitude. In the input data block, thicknesses of each layer in mm are required to fill, as well as corresponding elastic modulus of each layer in both horizontal directions. Loading is taken into account by inputting density (self-weight) and load(imposed load). And finally length and width.

5.2 Status

If there is any input data missing at a certain cell, not only there will be a reminder in the next cell in red color, but also a long red bar in status block. When there exists a red bar, it means that the input data is not completed, thus the result will not be shown, or will be invalid.

5.3 Results

There will be the maximum deflection based on the input data, which is compared with the expected value of L/300. One cell below the result, the conclusion of whether the result is ok or not is shown in big red font. If there is any red bar in status block, the result will be 'Non'.

At the bottom of results block, a compilation of deflections of the the plate under different length and width combinations is shown.

5.4 Intermediate data

The intermediate data is not of interest for the user of the sheet, but import for the calculation process. In this block, moment of inertia in both X and Y direction, and EI_x and EI_y are shown, as well as results of interpolating process as discussed in section 4.4.6.

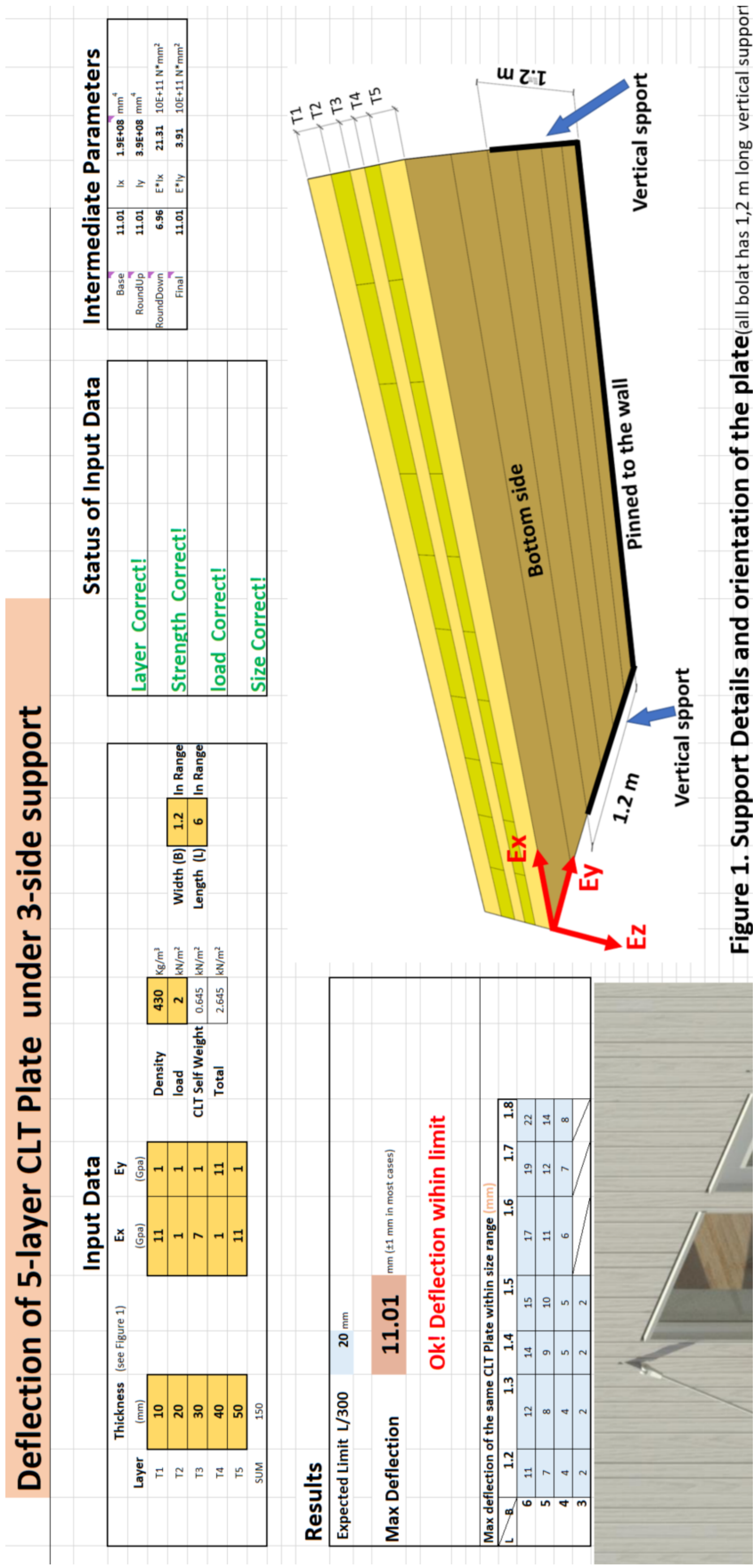


Figure 10: Layout of the Excel sheet for deflection.

6 Analysis and discussion

6.1 Accuracy

As shown in figure 2, the presented method of calculating maximum deflection is based on simulation in Abaqus and curve fitting process, meaning that it is suspected to deviations.

In some processes, such as in table 13, the fitted curves were an average of all dots plotted. Each dot means a specific case in reality. Applying the curves in the calculation brings decent results with minor difference to FEM in some cases, but in other cases bigger deviations occur.

Generally speaking, the method reaches an accuracy within 0.5 mm deviation, as shown in table 15 or in figure 12. for L = 6 m, there are several cases where deviation is bigger than 1 mm, the reason for this needs to be further investigated. Those bigger deviations when L = 6 m is minimized to below 1 mm when L = 5 m, see 11.

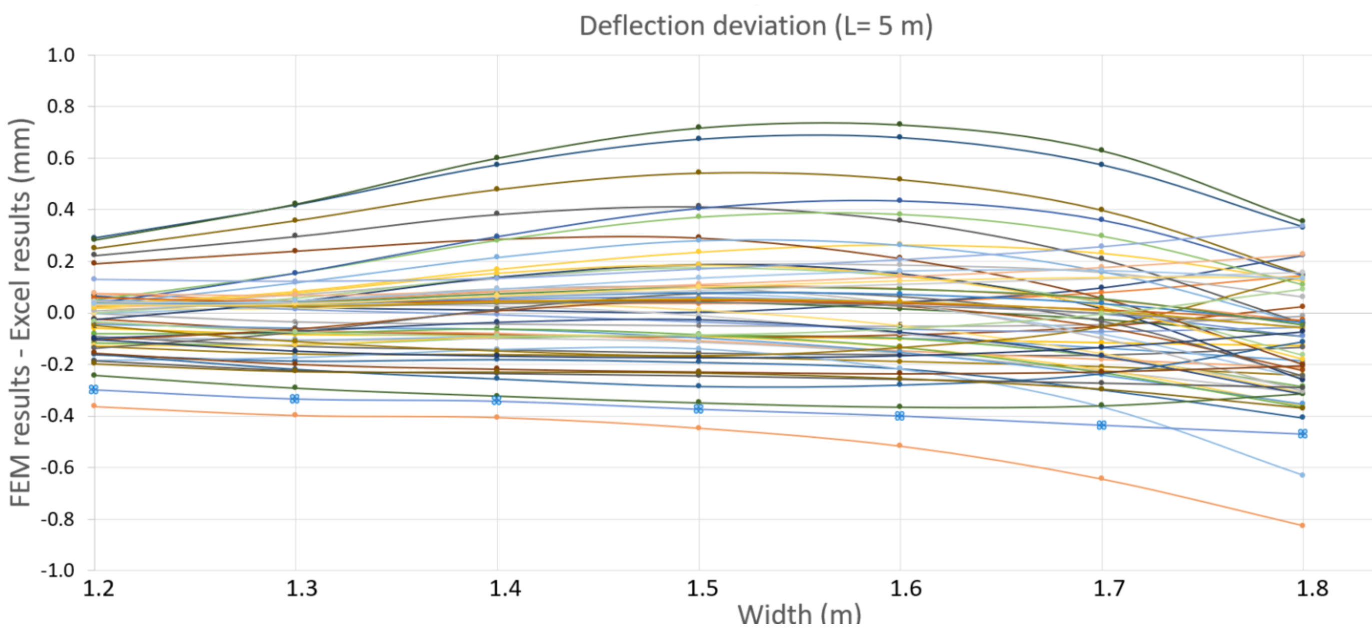


Figure 11: Deflection deviations based on table 15, when L = 5 m. Negative sign means the Excel-sheet-calculated deflection is bigger than FEM-calculated deflection.

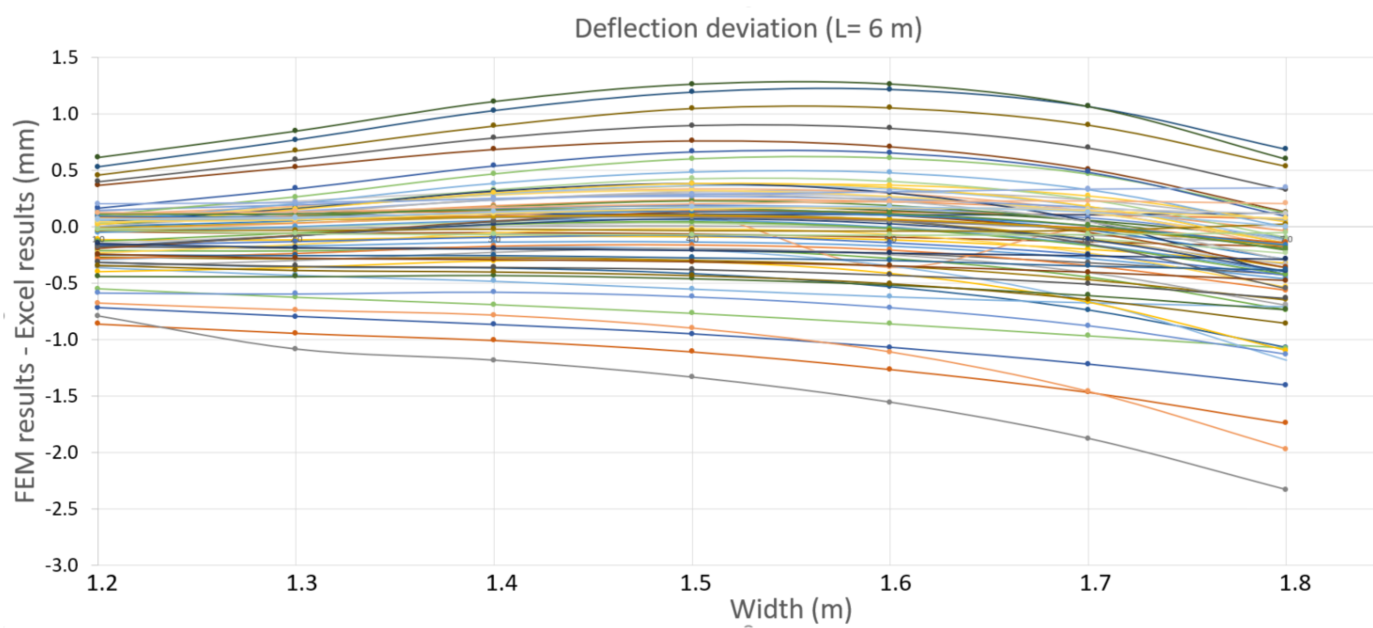


Figure 12: Deflection deviations based on table 15, when L = 6 m.

6.2 Range

Since different layer thickness combinations and layer strength combinations were taken into account when building the connections, this method is theoretically able to cover situations with any type of layer thickness combinations and strength combinations. It has been verified that for T=120 mm and 220 mm, the results are within 0,5 mm, see table 12. But further verification is need when thickness is smaller that 120 mm or bigger than 220 mm.

The length this method covers is 3-6 m, 1.2-1.8 m for width. Any sizes beyond the range may lead to lager deviations.

Table 12: Verification of the method when T=220 mm. During the connection-building, layer configurations were symmetrical in terms of thickness and strength. This example has proven the reliability of the method when layer configurations are asymmetrical.

T(mm)	**Ex**	**Ey**	**Size**	**FEM**	**Excel**	**Ex**	**Ey**	**Size**	**FEM**	**Excel**
20	11	1	6x1.2	4.62	4.60	**5**	1	6x1.2	5.95	5.90
60	1	11	6x1.5	6.42	6.68	1	11	6x1.5	8.74	8.79
40	11	1	4x1.75	3.14	3.34	**6**	1	4x1.75	4.79	5.01
30	1	11	3.5x1.45	1.55	1.70	1	11	3.5x1.45	2.39	2.53
70	11	1				**3**	1			

6.3 Source of error

This approach is by its nature an approximation of the analytic solution (if there exists such an analytic solution under this situation). The errors come from mainly several parts:

- As mentioned in figure 3, the support of CLT plate is idealized from the the real situation, leading to certain level of deviation. From the results of several verified cases, the difference is minor, but **a further verification is needed before using this method in real applications**.

- The FEM results were used as the bases to develop the method. Any error in FEM results will have an impact on the accuracy of the method. The mesh size is directly related to the accuracy of FEM results. The size of 0.004 m adopted for this project is believed to bring decent results. In the FEM simulation, the fixture at the wall-side was assumed to be pinned without any moment-bearing capacity; in reality the wall-side is fixed by steel brackets. This type of error exists with all FEM simulations.
- Some of the formulas (for example Equation 4) were developed as a balance between accuracy and ease of work. If taking EI_y into consideration, the result will be even closer to the FEM results, as figure 13 implies, at the expense of longer time and more effort.
- $W_{6\times1.2}$ was used as the base for $W_{6\times B}$; $W_{6\times B}$ was used as the base for $W_{5\times B}$ and in other lengths. It might happen that the deviation at $W_{6\times1.2}$ accumulates for example at $W_{3\times B}$. It should be mentioned that in the equation $W_{3\times B} = K_{6\times B} \cdot W_{6\times B}$, $K_{6\times B}$ is around 0.2, which means the deviation at $W_{6\times B}$ is somewhat minimized. Figure 12 has clearly showed this trend.
- Most of the relations built in the process are non-linear with few exceptions. To make the method able to calculate deflections of any length, interpolating was adopted (section 4.4.6), with the assumption that $W_{6\times B}$ is linearly related to $W_{5\times B}$, $W_{4\times B}$ and $W_{3\times B}$, which is not the case in reality.

6.4 Ways to improve

- The presented approach is just a numerical approach to the problem, a better way is to find a analytic way to solve the problem. But this would be a very difficult way.
- $W_{6\times B}$ was used as the base for deflection under other sizes. If, for example,$W_{5\times B}$ is decoupled from $W_{6\times B}$, the results of deflection when $L = 5\,m$ will be less affected by the errors at $W_{6\times B}$.
- If even more plate thicknesses are investigated and simulated with Abaqus, for example, when $T = 120\,mm$ and $T = 220\,mm$, the results obtained from the Excel sheet will be more close to the FEM results.
- If more time is allocated to the connection-building process, more variables can be taken into account (such as EI_y for $W_{6\times1.2}$), which will lead to a higher accuracy within the preset range.

7 Conclusion

The aim of the project is to create a simplified tool to calculate maximum deflection of CLT plates under 3-side-support. The presented method has achieved the goal. To get the maximum deflection, the users simply need to type in layer thicknesses, strength, load and size of the plate, without any specific knowledge about CLT or structural design.

The Excel sheet can be applied to the situations when the requirement for deflection is in a relatively lower precision (say 0.5 or 1 mm). This Excel sheet is able to cover deflection calculation of 5-layer CLT plate under 3-side-support, with the size from $6 \times 1.8\,m^2$ to $3 \times 1.2\,m^2$, and thickness between 120 mm and 220 mm despite of strength or thickness combinations of each layer.

The way of connection-building adopted in this report can be applied to find calculation method for deflections of CLT plates with intermediate supports or with other plate shapes.

References

[1] Tina Ersson. Balkonger i trähus: Systematisering av konstruktionsarbete, 2019.

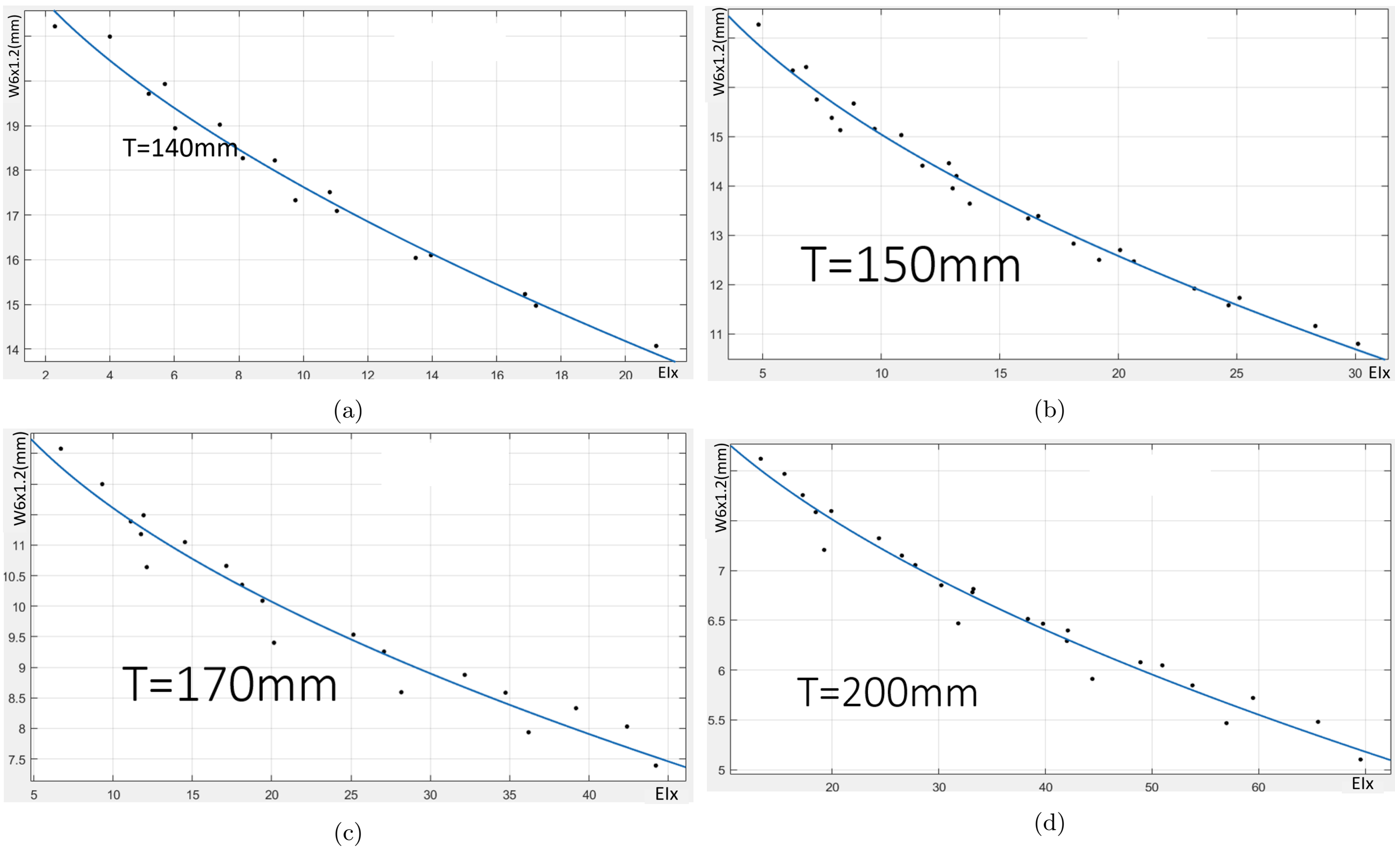
[2] Jiejun Wang, Fan Ning, Junzhu Li, and Houyuan Zhu. Experimental study and finite element simulation analysis of the bending properties of cross-laminated timber (clt) two-way plates. *Journal of Engineering Science & Technology Review*, 13(4), 2020.

A CLT ID numbers

Table 13: Some of CLT plates used in the project. Thickness T1-T5 in mm .

CLT No.	T1	T2	T3	T4	T5	Sum
1	20	40	20	40	20	140
2	20	30	40	30	40	160
3	30	40	30	40	30	170
4	40	20	30	20	40	150
5	40	30	30	30	40	170
6	40	40	40	40	40	200
7	50	20	30	20	50	170
8	20	50	30	50	20	170
9	10	60	30	60	10	170
10	60	10	30	10	60	170
20	10	50	20	50	10	140
22	30	30	20	30	30	140
21	50	10	20	10	50	140
30	30	30	30	30	30	150
31	20	40	30	40	20	150
32	10	50	30	50	10	150
33	50	10	30	10	50	150
40	20	60	40	60	20	200
41	30	50	40	50	30	200
42	50	30	40	30	50	200
43	60	20	40	20	60	200

B Curve-fitting for deflection with size $6 \times 1.2\,m^2$



(a) (b) (c) (d)

Figure 13: Curve-fitting process for 4 different thicknesses. Equation 1 was derived from those figures.

C Table for deflection with length of $5\,m$

D Verification

Table 14: Relations between $W_{5\times B}$ and $W_{5\times 1.2}$, gap means $W_{5\times B}$-$W_{5\times 1.2}$

	GPa	0	-0.00328	-0.00547	-0.00677	-0.00692	-0.00659	-0.008
	Average	0.649488	0.646205	0.644017	0.642719	0.642571	0.6429	0.644491
T(mm)	**CLT** No.	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**
170	5	0.625	0.621	0.619	0.617	0.618	0.620	0.625
		0.633	0.628	0.626	0.624	0.624	0.626	0.629
		0.642	0.638	0.635	0.633	0.633	0.634	0.636
		0.653	0.650	0.648	0.647	0.646	0.646	0.648
		0.669	0.667	0.666	0.665	0.666	0.666	0.667
	7	0.623	0.618	0.615	0.614	0.616	0.619	0.625
		0.630	0.625	0.622	0.621	0.622	0.625	0.630
		0.640	0.635	0.633	0.631	0.631	0.633	0.637
		0.653	0.649	0.647	0.646	0.645	0.647	0.649
		0.669	0.667	0.666	0.666	0.666	0.667	0.669
	8	0.637	0.633	0.630	0.628	0.627	0.627	0.628
		0.643	0.640	0.637	0.635	0.634	0.634	0.634
		0.651	0.648	0.646	0.644	0.643	0.642	0.643
		0.661	0.658	0.656	0.656	0.654	0.654	0.654
		0.673	0.672	0.671	0.671	0.670	0.671	0.671
140	1	0.630	0.624	0.621	0.618	0.616	0.616	0.617
		0.637	0.632	0.629	0.626	0.624	0.623	0.623
		0.645	0.641	0.638	0.636	0.634	0.633	0.633
		0.655	0.652	0.650	0.648	0.647	0.646	0.645
		0.668	0.667	0.665	0.665	0.664	0.664	0.664
	20	0.644	0.640	0.637	0.635	0.632	0.631	0.630
		0.650	0.647	0.644	0.642	0.640	0.638	0.637
		0.657	0.654	0.652	0.650	0.648	0.647	0.646
		0.665	0.663	0.661	0.660	0.659	0.658	0.657
		0.674	0.672	0.672	0.672	0.671	0.671	0.671
	22	0.623	0.617	0.613	0.611	0.610	0.611	0.614
		0.630	0.625	0.621	0.619	0.630	0.618	0.620
		0.640	0.635	0.632	0.629	0.628	0.628	0.628
		0.651	0.648	0.645	0.643	0.642	0.641	0.642
		0.667	0.664	0.663	0.662	0.661	0.661	0.662
150	31	0.633	0.628	0.625	0.622	0.621	0.620	0.621
		0.640	0.636	0.633	0.630	0.629	0.628	0.628
		0.648	0.645	0.642	0.640	0.638	0.637	0.637
		0.659	0.655	0.653	0.652	0.650	0.650	0.650
		0.671	0.669	0.668	0.668	0.668	0.667	0.667
	33	0.618	0.612	0.608	0.607	0.609	0.612	0.619
		0.627	0.621	0.618	0.616	0.617	0.620	0.625
		0.638	0.633	0.629	0.628	0.628	0.629	0.633
		0.652	0.648	0.645	0.643	0.643	0.644	0.647
		0.668	0.666	0.665	0.665	0.665	0.666	0.668
	32	0.648	0.645	0.642	0.640	0.638	0.636	0.636
		0.654	0.651	0.649	0.647	0.645	0.643	0.643
		0.660	0.658	0.656	0.655	0.653	0.652	0.651
		0.668	0.666	0.665	0.664	0.663	0.662	0.662
		0.676	0.676	0.675	0.675	0.675	0.675	0.675

Table 15: Verification of the method when T=170 mm and 140 mm.

			Comparison L=6 m (FEM - Calculation)							Comparison L=5 m (FEM - Calculation)						
T(mm)	**CLT** No.	Ex	1.2	1.3	1.4	1.5	1.6	1.7		1.2	1.3	1.4	1.5	1.6	1.7	1.8
170	5	11	0.1	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
		9	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1
		7	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.3
		5	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.1	-0.1	-0.2	-0.3	-0.5
		3	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.3	-0.2	-0.2	-0.2	-0.2	-0.3	-0.4	-0.7
170	7	11	0.1	0.0	0.0	0.0	0.0	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1
		9	0.0	-0.1	-0.1	-0.1	-0.1	-0.1	0.0	0.0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2
		7	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.4
		5	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.5	-0.6
		3	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.4	-0.3	-0.4	-0.4	-0.4	-0.5	-0.7	-1.1
170	8	11	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	0.1	0.2	0.2	0.1	0.1	-0.1
		9	0.0	0.0	0.1	0.1	0.0	0.0	-0.1	0.1	0.1	0.2	0.2	0.2	0.1	-0.1
		7	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	0.1	0.2	0.2	0.2	0.1	0.0
		5	0.0	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.3	0.3	0.2	0.1
		3	0.0	0.1	0.2	0.2	0.3	0.2	0.1	0.1	0.2	0.3	0.4	0.4	0.3	0.0
170	10	11								-0.4	-0.4	-0.5	-0.5	-0.6	-0.7	-0.7
		9								-0.5	-0.6	-0.7	-0.8	-0.9	-1.0	-1.1
		7								-0.7	-0.8	-0.9	-1.0	-1.1	-1.2	-1.4
		5								-0.9	-0.9	-1.0	-1.1	-1.3	-1.5	-1.7
		3								-0.8	-1.1	-1.2	-1.3	-1.6	-1.9	-2.3
170	9	11								0.0	0.0	0.1	0.1	0.1	-0.1	-0.4
		9								0.0	0.1	0.1	0.2	0.1	0.0	-0.3
		7								0.0	0.1	0.2	0.2	0.2	0.1	-0.2
		5								0.1	0.1	0.2	0.3	0.3	0.1	-0.1
		3								0.1	0.2	0.3	0.3	0.3	0.1	-0.2
140	1	11	0.0	0.0	0.0	0.0	-0.1	-0.2	-0.3	0.1	0.2	0.3	0.3	0.3	0.1	-0.1
		9	0.0	0.0	0.0	0.0	0.0	-0.2	-0.3	0.1	0.2	0.3	0.3	0.3	0.1	-0.1
		7	0.0	0.0	0.1	0.1	0.0	-0.1	-0.3	0.0	0.2	0.3	0.4	0.3	0.2	-0.1
		5	0.0	0.1	0.1	0.2	0.1	0.0	-0.2	0.0	0.2	0.3	0.4	0.4	0.2	-0.1
		3	0.0	0.0	0.1	0.2	0.2	0.0	-0.3	0.0	0.2	0.3	0.4	0.3	0.1	-0.4
140	20	11	0.2	0.2	0.3	0.3	0.2	0.1	-0.2	0.4	0.5	0.7	0.8	0.7	0.5	0.1
		9	0.2	0.3	0.4	0.4	0.4	0.2	0.0	0.4	0.6	0.8	0.9	0.9	0.7	0.3
		7	0.3	0.4	0.5	0.5	0.5	0.4	0.1	0.5	0.7	0.9	1.0	1.1	0.9	0.5
		5	0.3	0.4	0.6	0.7	0.7	0.6	0.3	0.5	0.8	1.0	1.2	1.2	1.1	0.7
		3	0.3	0.4	0.6	0.7	0.7	0.6	0.4	0.6	0.8	1.1	1.3	1.3	1.1	0.6
140	22	11	0.1	0.0	0.0	0.0	-0.1	-0.1	-0.1	0.1	0.2	0.3	0.3	0.2	0.2	0.1
		9	0.0	-0.1	-0.1	-0.1	-0.1	-0.2	-0.2	0.0	0.0	0.1	0.1	-0.4	0.0	-0.2
		7	-0.1	-0.1	-0.1	-0.1	-0.2	-0.2	-0.3	-0.1	-0.1	0.0	0.0	0.0	-0.1	-0.3
		5	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.4	-0.2	-0.2	-0.1	0.0	-0.1	-0.2	-0.5
		3	-0.2	-0.2	-0.1	-0.1	-0.2	-0.4	-0.6	-0.3	-0.2	-0.2	-0.2	-0.3	-0.7	-1.2

Table 16: Verification of the method when T=150 mm and 200 mm.

			Comparison L=6m (FEM - Calculation)							Comparison L=5m (FEM - Calculation)						
T(mm)	**CLT** No.	Ex	1.2	1.3	1.4	1.5	1.6	1.7		1.2	1.3	1.4	1.5	1.6	1.7	1.8
150	5	11								0.0	0.0	0.0	0.0	0.0	-0.1	-0.2
		9								-0.2	-0.1	-0.1	-0.1	-0.1	-0.2	-0.4
		7								-0.3	-0.2	-0.2	-0.2	-0.2	-0.3	-0.6
		5								-0.4	-0.3	-0.2	-0.2	-0.2	-0.4	-0.7
		3								-0.4	-0.4	-0.3	-0.3	-0.4	-0.7	-1.1
150	31	11	0.0	-0.1	-0.1	-0.1	-0.2	-0.2	-0.4	-0.1	0.0	0.1	0.1	0.0	-0.2	-0.4
		9	-0.1	-0.1	-0.1	-0.1	-0.1	-0.2	-0.4	-0.1	-0.1	0.0	0.0	0.0	-0.2	-0.4
		7	-0.1	-0.1	0.0	0.0	-0.1	-0.2	-0.3	-0.2	-0.1	0.0	0.1	0.0	-0.1	-0.4
		5	-0.1	-0.1	0.0	0.0	0.0	-0.1	-0.2	-0.2	-0.1	0.0	0.1	0.1	-0.1	-0.4
		3	-0.1	-0.1	0.0	0.1	0.1	0.0	-0.2	-0.2	-0.1	0.0	0.1	0.1	-0.1	-0.5
150	33	11	0.0	-0.1	-0.1	-0.2	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		9	-0.2	-0.2	-0.3	-0.3	-0.3	-0.2	-0.1	-0.2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.4
		7	-0.2	-0.3	-0.3	-0.3	-0.4	-0.4	-0.3	-0.4	-0.4	-0.4	-0.5	-0.5	-0.6	-0.7
		5	-0.3	-0.3	-0.3	-0.4	-0.4	-0.4	-0.5	-0.6	-0.6	-0.6	-0.6	-0.7	-0.9	-1.1
		3	-0.4	-0.4	-0.4	-0.4	-0.5	-0.6	-0.8	-0.7	-0.7	-0.8	-0.9	-1.1	-1.5	-2.0
150	32	11	0.0	0.0	0.1	0.1	0.0	-0.1	-0.3	0.0	0.1	0.2	0.3	0.2	0.0	-0.3
		9	0.0	0.1	0.2	0.2	0.1	0.0	-0.2	0.0	0.2	0.3	0.4	0.3	0.2	-0.2
		7	0.0	0.1	0.2	0.3	0.3	0.2	0.0	0.0	0.2	0.4	0.5	0.5	0.3	0.0
		5	0.1	0.2	0.3	0.4	0.4	0.3	0.1	0.1	0.3	0.5	0.6	0.6	0.5	0.1
		3	0.0	0.2	0.3	0.4	0.4	0.4	0.1	0.2	0.3	0.5	0.7	0.7	0.5	0.1
200	40	11	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	-0.1
		9	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	0.1	0.1	0.1	0.0	0.0	-0.2
		7	0.0	0.0	0.0	0.1	0.0	0.0	-0.1	0.1	0.1	0.1	0.1	0.1	0.0	-0.2
		5	0.0	0.0	0.1	0.1	0.1	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	-0.2
		3	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.2	0.2	0.1	0.0	-0.2
200	6	11	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.2	0.2	0.2	0.3	0.3	0.3	0.3
	6	9	0.1	0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.1	0.2	0.2	0.2	0.2	0.2
	6	7	0.0	0.0	0.1	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.2	0.2	0.1
	6	5	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.1	0.1	0.2	0.2	0.1	0.1
	6	3	0.0	0.0	0.1	0.1	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.0
200	43	11	0.0	-0.1	-0.1	-0.1	-0.1	0.0	0.1	0.0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
		9	-0.1	-0.1	-0.2	-0.2	-0.2	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.3	-0.3
		7	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.4	-0.5
		5	-0.2	-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.4	-0.4	-0.4	-0.5	-0.6
		3	-0.2	-0.2	-0.2	-0.2	-0.3	-0.3	-0.4	-0.3	-0.4	-0.4	-0.4	-0.5	-0.6	-0.9
200	41	11								0.2	0.2	0.2	0.3	0.3	0.3	0.2
		9								0.1	0.1	0.2	0.2	0.2	0.2	0.2
		7								0.1	0.1	0.2	0.2	0.2	0.2	0.1
		5								0.1	0.1	0.2	0.3	0.3	0.2	0.2
		3								0.1	0.2	0.3	0.3	0.3	0.3	0.2